

Proposed problem to AMM

Evaluate the limit as $\varepsilon \rightarrow 0^+$ of

$$\int_0^{\frac{\pi}{4}-\varepsilon} \frac{a - \ln \cos x}{\cos(2x)(x^2 + (a - \ln \cos x)^2)} dx + \int_{\frac{\pi}{4}+\varepsilon}^{\frac{\pi}{2}} \frac{a - \ln \cos x}{\cos(2x)(x^2 + (a - \ln \cos x)^2)} dx$$